Integrerend project systeemtheorie

16/04/2013, Tuesday, 9:00-12:00

1 $(3 + 12 = 15)$ **Linearization**

Consider the state-space system

$$\begin{aligned}\dot{x}_1 &= -x_1^6 - x_2\\ \dot{x}_2 &= x_1 + u\\ y_1 &= x_2^2\\ y_2 &= x_1.\end{aligned}$$

(a) Show that $x_1(t) = -1$, $x_2(t) = -1$ and $u(t) = 1$ is a solution of the system.

(b) Determine the linearized system around this solution.

2 (20) **Kharitonov's theorem**

Consider the interval polynomial $p(s, a, b, \mu) = s^3 + s^2 + [a, b]s + [\mu a, \mu b]$ where a, b, μ are real numbers with $a < b$ and $0 < \mu$. For which values of a, b, and μ, does $p(s, a, b, \mu)$ have all its roots in the open left half-plane?

3 $(3 + 4 + 4 + 4 + 4 + 10 + 6 = 35)$ **Controllability and observability**

Consider the linear system

$$\dot{x} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} x.$$

Explain your answers to the following questions:

(a) Is it stable?

(b) Is it controllable?

(c) Is it observable?

(d) Is it stabilizable?

(e) Is it detectable?

(f) Does there exist a stabilizing state feedback $u = Fx$? If yes, determine such a feedback.

(g) Does there exist an observer of the form $\dot{\hat{x}} = P\hat{x} + Qu + Ry$? If yes, determine such a compensator.

4 $(3 + 17 = 20)$ **Companion form**

Consider the linear system

$$\dot{x}(t) = Ax(t) + bu(t)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the input, $A \in \mathbb{R}^{n\times n}$, and $b \in \mathbb{R}^n$. Suppose that (A, b) is controllable and $\det(\lambda I - A) = \lambda^n + p_{n-1}\lambda^{n-1} + \cdots + p_1\lambda + p_0$. Let the vectors $q_1, q_2, \ldots, q_n$ be defined as

$$\begin{aligned}
q_n &= b \\
q_{n-1} &= Ab + p_{n-1}b = Aq_n + p_{n-1}q_n \\
q_{n-2} &= A^2b + p_{n-1}Ab + p_{n-2}b = Aq_{n-1} + p_{n-2}q_n \\
&\vdots \\
q_1 &= A^{n-1}b + p_{n-1}A^{n-2}b + \cdots + p_2Ab + p_1b = Aq_2 + p_1q_n.
\end{aligned}$$

(a) Show that the vectors $q_1, q_2, \ldots, q_n$ are linearly independent.

(b) Let $T = \begin{bmatrix} q_1 & q_2 & \cdots & q_n \end{bmatrix}$. Show that

$$T^{-1}b = T^{-1}q_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and

$$T^{-1}AT = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -p_0 & -p_1 & -p_2 & \cdots & -p_{n-1} \end{bmatrix}.$$

(Hint: First find out AT.)

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10 pts gratis.